SPIN EFFECTS FOR NEUTRINOS AND ELECTRONS MOVING IN DENSE MATTER

A.V. Grigoriev [1], A.M. Savochkin [2], A.I. Studenikin[2†] and A.I. Ternov[3]

(1) *Skobeltsin Institute of Nuclear Physics, Moscow State University*

(2) *Department of Theoretical Physics, Moscow State University*

(3) *Department of Theoretical Physics, Moscow Institute for Physics and Technology*

† *E-mail: studenik@srd.sinp.msu.ru*

Abstract

We shortly summarize the present status of neutrino magnetic moment studies (theory and experiment). Then we discuss the quasiclassical treatment of neutrino spin evolution in matter. After that we come to the quantum approach to description of neutrino and electron motion in matter on the basis of the quantum wave equations exact solutions method with special focus on spin effects.

This paper is devoted to the problem of neutrino and electron motion in a dense matter with special focus on the spin phenomena.

It has been proven in recent oscillation experiments that neutrino has nonzero mass. Therefore, the Dirac neutrino should have nontrivial electromagnetic properties, in particular, nonzero magnetic moment. It is also well known [1] that in the minimally extended Standard Model with $SU(2)$-singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to the neutrino mass $\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu = 3 \times 10^{-19} \mu_0 \left(\frac{m_\nu}{1\text{eV}} \right)$, where $\mu_0 = e/2m$ is the Bohr magneton, m_ν and m are the neutrino and electron masses. There are also models (see [2]) in which much large values for magnetic moment of neutrino are predicted.

The LEP data require that the number of light neutrinos coupling to Z boson is exactly three, whereas any additional neutrino, if this particle exist, must be heavy. In light of this opportunity we considered the neutrino magnetic moment for various ratios of particles masses. We have obtained [3] values of the neutrino magnetic moment for light (for this particular case see also [1, 4]), intermediate and heavy massive neutrino: 1) $\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3(2-7a+6a^2-2a^2\ln a - a^3)}{4(1-a)^3}$, for $m_\nu \ll m_\ell \ll M_W$, 2) $\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\}$, for $m_\ell \ll m_\nu \ll M_W$, 3) $\mu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu$, for $m_\ell \ll M_W \ll m_\nu$, where $a = (\frac{m_l}{M_W})^2$ and $b = (\frac{m_\nu}{M_W})^2$. It should be also mentioned that the neutrino magnetic moment can be affected by the external environment. In particular, the value of the neutrino magnetic moment can be significantly shifted by the presence of strong external magnetic fields [5] (see also [6, 7]).

So far, solar neutrino experiments set a limit on the neutrino magnetic moment on the level of $\mu_{\nu_e} \leq 1.5 \times 10^{-10}$[8]. More stringent constraint $\mu_{\nu_e} \leq 5.8 \times 10^{-11}$ has been provided by the GEMMA accelerator experiment [9]. The constraint from astrophysical considerations (the red giants cooling) is $\mu_{\nu_e} \leq 3 \times 10^{-12}$ [10]. There are also constraints coming from estimations based on supernovae core collapse and primordial nucleosynthesis: $\mu_\nu \leq 3 \times 10^{-10}$ (see [11] and references therein).

Developing of the theory of neutrino spin properties in an external environment we have evaluated the Loretz invariant approach to the neutrino spin evolution that was

based on the proposed generalized Bargmann-Michel-Telegdi equation [12]. Within the developed Lorentz invariant approach it is also possible to find the solution for the neutrino spin evolution problem for a general case when the neutrino is subjected to general types of non-derivative interactions with external fields [13]. These interactions are given by the Lagrangian

$$-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t'}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \tag{1}$$

where $s, \pi, V^\mu = (V^0, \mathbf{V}), A^\mu = (A^0, \mathbf{A}), T_{\mu\nu} = (\mathbf{a}, \mathbf{b}), \Pi_{\mu\nu} = (\boldsymbol{c}, \boldsymbol{d})$ are the scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields, respectively. For the corresponding spin evolution equation we have found

$$\begin{gathered}\frac{d\mathbf{S}}{dt} = 2g_a\left\{A^0[\mathbf{S}\times\boldsymbol{\beta}] - \frac{(\mathbf{A}\boldsymbol{\beta})[\mathbf{S}\times\boldsymbol{\beta}]}{1+\gamma^{-1}} - \frac{1}{\gamma}[\mathbf{S}\times\mathbf{A}]\right\} + 2g_t\left\{[\mathbf{S}\times\mathbf{b}] - \frac{(\boldsymbol{\beta}\mathbf{b})[\mathbf{S}\times\boldsymbol{\beta}]}{1+\gamma^{-1}} + [\mathbf{S}\times[\mathbf{a}\times\boldsymbol{\beta}]]\right\} \\ +2ig_t'\left\{[\mathbf{S}\times\mathbf{c}] - \frac{(\boldsymbol{\beta}\mathbf{c})[\mathbf{S}\times\boldsymbol{\beta}]}{1+\gamma^{-1}} - [\mathbf{S}\times[\mathbf{d}\times\boldsymbol{\beta}]]\right\}.\end{gathered} \tag{2}$$

This is a rather general equation for the neutrino spin evolution that can be also used for description of neutrino spin oscillations in different environments such as moving and polarized matter with external electromagnetic fields (see [14, 15]).

Considering the neutrino spin evolution within the quasi-classical treatment on the basis of the above mentioned generalized Bargmann-Michel-Telegdi equation, we have predicted [16] a new mechanism for the electromagnetic radiation by a neutrino moving in the background matter. We have termed this radiation the "spin light of neutrino" ($SL\nu$) in matter [16]. The term "spin light" was used [17] for designation of the magnetic-dependent term in the radiation of an electron in a magnetic field. The $SL\nu$ effect also studied in the cases when electromagnetic and gravitational fields also present in matter [18]. Here we should like to mention that the considered $SL\nu$ is indeed a new type of electromagnetic radiation of a neutrino that can be emitted by the particle in matter. This radiation mechanism has never been considered before. As it was mentioned in our first papers on this subject [16], the $SL\nu$ in matter can not be considered as the neutrino Cherenkov radiation in matter because it can exist even when the emitted photon refractive index is equal to unit. The $SL\nu$ radiation is due to radiation of the neutrino by its own rather then radiation of the background particles.

As it was clear from the very beginning [16], the $SL\nu$ is a quantum phenomenon by its nature and later on we elaborated [19] the quantum theory of this radiation (see also [20]). To put it on a solid ground, we of have elaborated a rather powerful method that implies the use of the exact solutions of the modified Dirac equation for the neutrino wave function in matter.

Recently we have spread this developed method of the "exact solutions" to description of an electron moving matter [21, 22, 23] and derived the modified Dirac equation for an electron moving in matter and found its solutions. On the basis of this exact solution of this equation we have considered a new mechanism for the electromagnetic radiation that can be emitted by an electron in the background matter. This mechanism is similar to the $SL\nu$ in matter and we termed it the "spin light of electron" in matter [21].

As it was shown in [19, 21, 22, 23], in the case of the standard model interactions of electron neutrinos and electrons with matter composed of neutrons, the corresponding

modified Dirac equations for each of the particles can be written in the following form:

$$\Big\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(c_l+\gamma_5)\widetilde{f}^\mu - m_l\Big\}\Psi^{(l)}(x) = 0, \tag{3}$$

where for the case of neutrino $m_l = m_\nu$ and $c_l = c_\nu = 1$, whereas for electron $m_l = m_e$ and $c_l = c_e = 1 - 4\sin^2\theta_W$. For unpolarized matter $\widetilde{f}^\mu = \frac{G_F}{\sqrt{2}}(n_n, n_n\mathbf{v})$, n_n and $\mathbf{v}$ are, respectively, the neutron number density and overage speed. The solutions of these equations are as follows,

$$\Psi^{(l)}_{\varepsilon,\mathbf{p},s}(\mathbf{r},t) = \frac{e^{-i(E^{(l)}_\varepsilon t - \mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1+\frac{m_l}{E^{(l)}_\varepsilon - c\alpha_n m_l}}\ \sqrt{1+s\frac{p_3}{p}} \\ s\sqrt{1+\frac{m_l}{E^{(l)}_\varepsilon - c\alpha_n m_l}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta} \\ s\varepsilon\eta\sqrt{1-\frac{m_l}{E^{(l)}_\varepsilon - c\alpha_n m_l}}\ \sqrt{1+s\frac{p_3}{p}} \\ \varepsilon\eta\sqrt{1-\frac{m_l}{E^{(l)}_\varepsilon - c\alpha_n m_l}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta} \end{pmatrix}. \tag{4}$$

where the energy spectra are

$$E^{(l)}_\varepsilon = \varepsilon\eta\sqrt{\mathbf{p}^2\Big(1 - s\alpha_n\frac{m_l}{p}\Big)^2 + m^2} + c_l\alpha_n m_l, \quad \alpha_n = \pm\frac{1}{2\sqrt{2}}G_F\frac{n_n}{m_l}. \tag{5}$$

Here p, s and ε are the particles momenta, helicities and signs of energy, "$\pm$" corresponds to e and ν_e. The value $\eta = \mathrm{sign}(1 - s\alpha_n\frac{m_l}{p})$ is introduced to provide a proper behavior of the neutrino wave function in the hypothetical massless case.

It should be pointed out that the derived modified Dirac equations for a neutrino and electron in matter and their exact solutions obtained establish an effective method for investigation of different phenomena that can arise when the particles move in dense media (for more details see [22]), including the cases peculiar for astrophysical and cosmological environments. For example, effects of the Dirac neutrino reflection and trapping, as well as neutrino-antineutrino annihilation and neutrino pair creation in matter at the interface between two media with different densities can be considered on this basis (see [24] and references therein).

Using the exact solutions of the above mentioned Dirac equations for a neutrino and electron we have performed detailed investigations of the $SL\nu$ and SLe in matter. In particular, in the case of ultra-relativistic neutrinos ($p \gg m$) and a wide range of the matter density parameter α for the total rate of the $SL\nu$ we obtained [19]

$$\Gamma_{SL\nu} = 4\mu_\nu^2\alpha^2 m_\nu^2 p, \qquad m_\nu/p \ll \alpha \ll p/m_\nu. \tag{6}$$

The main properties of the $SL\nu$ investigated in [16, 18, 19] can be summarized as follows: 1) a neutrino with nonzero mass and magnetic moment when moving in dense matter can emit spin light; 2) in general, $SL\nu$ in matter is due to the dependence of the neutrino dispersion relation in matter on the neutrino helicity; 3) the $SL\nu$ radiation rate and power depend on the neutrino magnetic moment and energy, and also on the matter density; 4) the matter density parameter α, that depends on the type of neutrino and matter composition, can be negative; therefore the types of initial and final neutrino (and antineutrino) states, conversion between which can effectively produce the $SL\nu$ radiation,

are determined by the matter composition; 5) the $SL\nu$ in matter leads to the neutrino-spin polarization effect; depending on the type of the initial neutrino (or antineutrino) and matter composition the negative-helicity relativistic neutrino (the left-handed neutrino ν_L) is converted to the positive-helicity neutrino (the right-handed neutrino ν_R) or vice versa; 6) the obtained expressions for the $SL\nu$ radiation rate and power exhibit non-trivial dependence on the density of matter and on the initial neutrino energy; the $SL\nu$ radiation rate and power are proportional to the neutrino magnetic moment squared which is, in general, a small value and also on the neutrino energy, that is why the radiation discussed can be effectively produced only in the case of ultra-relativistic neutrinos; 7) for a wide range of matter densities the radiation is beamed along the neutrino momentum, however the actual shape of the radiation spatial distribution may vary from projector-like to cap-like, depending on the neutrino momentum-to-mass ratio and the matter density; 8) in a wide range of matter densities the $SL\nu$ radiation is characterized by total circular polarization; 9) the emitted photon energy is also essentially dependent on the neutrino energy and matter density; in particular, in the most interesting for possible astrophysical and cosmology applications case of ultra-high energy neutrinos, the average energy of the $SL\nu$ photons is one third of the neutrino momentum. Considering the listed above properties of the $SL\nu$ in matter, we argue that this radiation can be produced by high-energy neutrinos propagating in different astrophysical and cosmological environments.

Performing the detailed study of the SLe in neutron matter [23] we have found for the total rate

$$\Gamma_{SLe} = e^2 m_e^2/(2p)\left[\ln\left(4\alpha_n p/m_e\right) - 3/2\right], \qquad m_e/p \ll \alpha_n \ll p/m_e, \tag{7}$$

where it is supposed that $\ln \frac{4\alpha_n p}{m_e} \gg 1$. It was also found that for relativistic electrons the emitted photon energy can reach the range of gamma-rays. Furthermore, the electron can loose nearly the whole of its initial energy due to the SLe mechanism.

Several aspects of the background plasma effects in the $SL\nu$ radiation mechanism have been discussed in [19]. Recently this problem has been also considered in [25] and the total rates of the $SL\nu$ and SLe in plasma where derived. The final result of [25] for the $SL\nu$ rate, that accounted for the photon dispersion in plasma, in the case of ultra-high energy neutrino (i.e., when the time scale of the process can be much less than the age of the Universe) exactly reproduces our result (6) obtained in [19]. At the same time, the SLe total rate given by eq. (65) in the second paper of [25] in the leading logarithmic term confirms our result (7) obtained in [23].

Recently we have applied the developed method of exact solutions of quantum wave equations in the background matter to a particular case when a neutrino is propagating in a rotating medium of constant density [26]. Suppose that the neutrino propagates inside a uniformly rotating medium composed of neutrons. This can be considered for modelling of neutrino propagation inside a rotating neutron star. The corresponding modified Dirac equation for the neutrino wave function is given by (3) with the potential $\widetilde{f}^\mu$ that accounts for the medium rotation. The equation can be solved in the considered case and for the energy spectrum of the relativistic active left-handed neutrinos with vanishing mass we have obtained

$$p_0 = \sqrt{p_3^2 + 2\gamma N} - G_F n/\sqrt{2}, \quad \gamma = G_F \omega n/\sqrt{2}, \quad N = 0, 1, 2, ..., \tag{8}$$

where ω is the angular frequency of the star rotation. The energy depends on the neutrino momentum component p_3 along the rotation axis of matter and the quantum number N

that determines the value of the neutrino momentum in the orthogonal plane. Thus, it is shown that the transversal motion of an active neutrino is quantized very much like an electron energy is quantized in a constant magnetic field forming the Landau energy levels. From these properties of the neutrino energy spectrum we predict that there is an effect of trapping neutrinos with the correspondent energies inside rotating dense stars.

The two of the authors (A.G. and A.S.) are thankful to Anatoly Efremov and Oleg Teryaev for the invitation to attend the XII Workshop on High Energy Spin Physics and for the kind hospitality provided in Dubna.

References

[1] K. Fujikawa, R. Shrock, Phys.Rev.Lett. **45**, 963 (1980).

[2] J. Kim, Phys.Rev. **D14**, 3000 (1976); M. Beg, W. Marciano, Phys.Rev. **D17**, 1395 (1978); M. Voloshin, M. Vysotsky, L. Okun, Sov.Phys.JETP **64**, 446 (1986); M. Fukugita, T. Yanagida, Phys.Rev.Lett. **58**, 1807 (1987); M. Voloshin, Sov.J.Nucl.Phys. **48**, 512 (1988).

[3] M. Dvornikov, A. Studenikin, Phys.Rev. **D69**, 073001 (2004); JETP **99** (2004) 254.

[4] L. Cabral-Rosetti, J. Bernabeu, J. Vidal, A. Zepeda, Eur.Phys.J. **C 12**, 633 (2000).

[5] A. Borisov, V. Zhukovsky, V. Kurilin, A. Ternov, Sov.J.Nucl.Phys. **41**, 743 (1985).

[6] A. Egorov, G. Likhachev, A. Studenikin, in: Elementary Particle Physics, Publ. by ICAS (Moscow, Russia, 1999), 52.

[7] J. Nieves, Phys.Rev. **D68**, 113003 (2003).

[8] J. F. Beacom and P. Vogel, Phys. Rev. Lett. **83**, 5222 (1999); D. W. Liu *et al.*, Phys. Rev. Lett. **93**, 021802 (2004).

[9] A.Beda et al, Phys.Atom.Nucl. **70**, 1873 (2007).

[10] G. G. Raffelt, Phys. Rep. **320**, 319 (1999).

[11] A. Ayala et al, Nucl.Phys.B **564**, 204 (2000); A. Ayala et al, Phys.Rev.D **59**, 111901 (1999).

[12] A.Egorov,A.Lobanov,A.Studenikin,Phys.Lett. B491 (2000) 137; A.Lobanov, A.Studenikin, Phys.Lett.B **515**, 94 (2001);

[13] M.Dvornikov, A.Studenikin, JHEP **09** (2002) 016.

[14] A. Studenikin, *Neutrinos in magnetic fields: from the first studies to the new effects in neutrino oscillations*, in: Results and Perspectives in Particle Physics, Les Recontres de Physique de la Vallee d'Aoste, ed. by M.Greco, Frascati Physics Series, V.XXXIV, p. 155; Nucl.Phys. B (Proc.Suppl.) **143**, 570 (2005); hep-ph/0407010.

[15] A. Studenikin, Phys.Atom.Nucl. **70**, 1275 (2007).

[16] A. Lobanov, A. Studenikin, Phys.Lett.B **564**, 27 (2003), hep-ph/0212393; Phys.Lett.B **601**, 171 (2004) .

[17] I.M.Ternov, Sov.Phys.Usp. **38**, 405 (1995); V.A.Bordovitsyn, I.M.Ternov, V.G.Bagrov, Sov.Phys.Usp. **38**, 1037 (1995).

[18] M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys.D **14**, 309 (2005) .

[19] A. Studenikin, A. Ternov, Phys.Lett. B **608**, 107 (2005), hep-ph/0410297, hep-ph/0410296; A. Grigoriev, A. Studenikin, A. Ternov, Phys.Lett. B **622**, 199 (2005), hep-ph/0502231; A. Grigoriev, A. Studenikin, A. Ternov, Grav. & Cosm. **11**, 132 (2005).

[20] A.Lobanov, Dokl.Phys. **50**, 286 (2005), hep-ph/0411342; Phys.Lett.B **619**, 136 (2005), hep-ph/0506007.

[21] A. Studenikin, J.Phys.A: Math.Gen. **39**, 6769 (2006).

[22] A. Studenikin, Ann.Fond. de Broglie **31**, no 2-3, (2006), hep-ph/0611100.

[23] A. Grigoriev, S. Shinkevich, A. Studenikin, A. Ternov, I. Trofimov, Spin light of electron in matter, in: Particle Physics at the Year of 250th Anniversary of Moscow University, ed. by A.Studenikin, World Scientific (Singapore), p. 73 (2006); Izv.Vuzov.Fizika (Russia) No.6, p.66 (2007), hep-ph/0611128.

[24] A. Grigoriev, A. Studenikin, A. Ternov, Phys.Atom.Nucl. **69**, 1940 (2006).

[25] A. Kuznetsov, N. Mikheev, Mod.Phys.Lett A **23**, 1769 (2006); Int.J.Mod.Phys. A **22** 3211 (2007).

[26] A. Grigoriev, A. Savochkin, A. Studenikin, Izv.Vuzov.Fizika (Russia) No. 8, 90 (2007).